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Queuing Models: Application to Kibagabaga Hospital Services

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Abstract: **The purpose of this project was to apply queuing models to KIBAGABAGA Hospital services and analyze time a patient can spend waiting for a service at this hospital. It was also to provide necessary information to policy makers aiming to contribute to the wellbeing of population by reducing waiting time for services. In excessive cases, long queues can delay appropriate decision for a specific disease that can cause occurrence of death while patient still wait for service. This project, examined the average time a patient spend waiting in the queue, time a patient can spend waiting in the system, average number of patient waiting in the queue and average number of patient waiting in the system by applying queuing models in 3 consultation rooms of KIBAGABAGA Hospital. We computed the system utilization and found that it is greater than one which means that the queue will grow without bound. There were a large number of patients waiting in the queue and they waited a long time before meeting a medical doctor. The correlation analysis proved that there was a negative correlation between patient arrival and working days which means that there were many patients on Monday than on Friday. To reduce the waiting time, we suggested that the hospital should increase the number of physician and nurses as well as the consultation rooms and to develop a staffing plan that will put more effort in the beginning of the week**.

Keywords: **Queuing model, waiting time, customer arrival, customer service, outpatient department.**

I. INTRODUCTION

1.1 statement of the problem:

A Queuing problem arises when the current service rate of facility falls short of the current service rate demands of customers**.** The service facilities whose customers are patients vary generally in capacity and size, from small outpatient clinics to large, urban hospitals to referral hospitals. Regardless these differences, healthcare processes can be categorized based on how patients arrive, wait for service, obtain service, and then depart. The servers in hospital queuing systems are the trained staff and equipment required for specific activities and procedures. In excessive cases, long queues can delay appropriate decision for a specific disease that can cause occurrence of death while patient still wait for service. Therefore, queuing has become a sign of incompetence of public hospitals in the world and Rwanda is not an exception. Decrease of waiting time of patients for healthcare service is one of the challenges facing the majority of hospitals. A few of the factors that is responsible for long waiting lines or delays in providing service are: lack of passion and commitment to work on the part of the hospital staff, overloading of available doctors, doctors attending to patients in more than one clinic etc (Belson, 1988).

These put doctors under stress and tension, hence tends to dispose off a patient without in-depth probing or treatment, which often leads to patient dissatisfaction (Babes, 1991). The resulting performance variables can be used by the policy makers to increase competence, improve the quality of patient care and reduce cost in hospital institutions as well.

1.2 Justification of the study:

This project was conducted in order to fulfill the requirements for the award of the degree of Master of Science in Applied Statistics and it will benefit in different ways: It will increase the knowledge of the student by relating the theories encountered from lectures to the real world of application. It will also contribute to increase patients' satisfaction in public health facilities. The decision makers in health system will benefit from this research by using results to develop their staffing plan. This research will serve as reference for other researchers in this field by filling the gaps encountered in present research.

1.3 Objectives:

1.3.1. General objective:

The general objective of this project was to apply queuing models to KIBAGABAGA Hospital services

1.3.2 Specific Objectives:

1. To determine the mean number of arrivals per hour (λ) in KIBAGABAGA hospital.
2. To find the mean number of patients served per hour (μ) in KIBAGABAGA hospital.
3. To calculate the average time a patient spends waiting in the queue before meeting a medical doctor in KIBAGABAGA hospital.
4. To calculate the average time a patient spends waiting in the system to get served.
5. To analyze the waiting line of patients at KIBAGABAGA hospital.

1.4. Research Questions:

1. What is the mean number of arrivals per hour (λ)?
2. What is the mean number of patients served per hour (μ)?
3. What is the average time a patient spends waiting before meeting a medical doctor?

1.5 Scope of the study:

This project was conducted to the patients visiting KIBAGABAGA hospital in Outpatient Department (OPD) for consultation in 23 working days only from Monday to Friday.

1.6 Limitation of the study:

Due to insufficiency funds and time constraints, this research was conducted only to the patients visiting KIBAGABAGA hospital in outpatient department for consultation from Monday to Friday.

II. METHODOLOGY

2.1 Research design:

In this project, the patients were coming from infinite population and the system was enough to receive all the patients coming in Outpatient department (OPD). There were three consultation rooms (3 servers) to receive patients (customers).

2.2 Target population:

Depelteau, F. (2000, p. 213) defines the population as being" *a set of all individuals who have precise characteristics in relationship with the objectives"*. For our case, the population of inquiry concerned the patients who were coming for consultation in Outpatient department in Kibagabaga hospital.

2.3 Sampling techniques:

Population sizes was considered either unlimited (essentially infinite) or limited (finite). When the number of patients or arrivals on hand at any given moment is just a small portion of all potential arrivals, the arrival population was considered unlimited, or infinite. In our case where patients arrive at KIBAGABAGA Hospital were considered to be unlimited (Heizer, Render, 2004).

In most statistical research studies, population parameters are usually unknown and have to be estimated from the sample. To determine the size of the sample how large or how small; therefore the sample should be of optimal size, not too large or not too small. Sample size should be large enough to give CI (confidence interval) of the desired width.

To determine the sample size one is required to specify the precision of the estimation desired. When a study is being made, sampling error arises and it is controlled by selecting a sample of adequate size. Usually, we use 95% of CI which is two tailed test (Augustin 2015)

$$n = \frac{z^2 p(1-p)}{E^2}$$

Where,

n=the desired sample size,

z=the standard normal deviate at the realized confidence interval/Hence level,

p=. Healthcare service utilization rate in KIBAGABAGA hospital which corresponds to 86%,

E= Margin of Error (Confidence Interval); in our case we have decided to use 0.04.

The table below shows different values used in calculations and the equivalent sample size (n)

Table 1: Sample size calculations

Z	P	1-P	E
1.96	0.86	0.14	0.05

$$\boldsymbol{n = \frac{(1.96)^2 * (0.86)(1-0.86)}{0.05^2} = 185\ hours}$$

Here we obtained a sample size of 185 hours that correspond to 23 days if we consider 8 working hours per day.

2.3.1 Sampling Strategy:

Simple random sampling strategy was used (probability sampling) since each patient in the population has an equal and independent chance of being selected. Data was collected during the 23 day (using the above formula). The arrival times of all patients as they arrive randomly was recorded, the time they start being served and eventually record the time they depart.

2.4 Instruments:

In our work, different documents were used such as books, reports and electronic sources. All these documents helped us to make the conceptual and theoretical framework of our work as well as to analyze the data and interpret the results. Also, we recorded discrete time for patient arrival and service.

2.5 Data Collection Procedure:

Data was collected for a period of 23 days from Monday to Friday, Observation technique was used as they arrive randomly, the time they start being served and eventually record the time they depart. This helped us to draw a table used in estimating the average number of patients entered in the system and average number of patients served in one hour. From this estimated the remaining performance parameters of the system.

2.6 Data Processing and Analysis:

For analysis of our data and interpretation of the results, different computer tools was used especially Microsoft Excel and SPSS. The data collected using observation technique was entered in Excel spread sheet for cleaning and convert the recorded time in interval time and then imported in Statistical Package for Social Sciences (SPSS) for analysis where descriptive statistics and significance test were carried out as well as estimation of different performance parameters describing the behavior of the system. The figures and tables were interpreted in scope predefined objectives in order to make data meaningful and come out with conclusions and recommendations.

The system performance parameters that were used in this study were defined as follows:

λ: Arrival rate of patients at outpatient department per hour;

μ: Service rate (Length of stay) of patients at outpatient department per hour;

c: Number of doctors (servers) working in outpatient department for consultation. In this model, there will be three parallel physicians.

ρ : Outpatient system utilization factor

$$\rho = \frac{\lambda}{c_{\mu}}$$

L_q: Average number of patients at outpatient department in the queue.

$$L_q = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2}$$

L: Average number of outpatients in the system

$$\mathrm{L} = L_q + \frac{\lambda}{\mu}$$

W_q: Waiting time of outpatients in the queue

$$W_q = \frac{L_q}{\lambda}$$

W: Waiting time of outpatients in the system

$$W = \frac{L}{\lambda}$$

P_n = probability of n outpatients existing in the system.

$$P_n = \begin{cases} \dfrac{\left(\dfrac{\lambda}{\mu}\right)^n}{n!} * P_o & \text{If } 0<n<c \\ \dfrac{\lambda^n}{c!\,\mu^n c^{n-c}} * P_o & \text{If } c \le n \end{cases}$$

P_0 = Possibility of 0 outpatients existing in the system.

$$P_o = \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \left(\frac{1}{1-\dfrac{\rho}{c}} \right) \right]^{-1}$$

III. RESEARCH FINDINGS

3.1 Mean Number of Arrivals per hour:

Table 2: Mean of interval time

	N	Min	Max	Mean	Std. Deviation
Arrival interval time	3121	0	41	3.98	4.41
Valid N	3121				

The mean number of arrivals has been calculated from the data collected during 23 days of field visit in KIBAGABAGA hospital in outpatient department. Time duration was recorded for each patient arriving for consultation in outpatient department, and then the interval time period separating a patient arrival and the next was calculated. Finally the average interval time was calculated. After these calculations we found that on average every **3.9894** minutes one patient joined the queue. This corresponds to **λ=15.0398** patients arrived per hour (1hour=60min)

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3.2 The mean number of patients served per hour (λ):

Table 3: Service interval time

	N	**Min**	**Max**	**Mean**	**Std. Deviation**
Arrival interval time	1042	0	51	6.0456	8.409
Valid N	1042				

This mean number of patient served per time period has been calculated based on the records of time a patient enter in the consultation room and the time the patient go out from the office.

These data have been collected in 23 working days and after we have calculated the time spent by each patient in this office. At the end we calculated the average time spent by a patient in the

Physician's office, we found that every 6.0456 minutes there was one patient served by all physicians together.

This means that every 18.1308 minutes there was one patient served by one physician; this corresponds to μ **=3.3092** patients served per hour by one physician.

3.3 System utilization factor (ρ):

The system utilization factor has been calculated using the mean number of arrivals and the mean number of patient served per hour to show that there is probability that the queue can be formed or not and also to have an idea on the system performance.

$$\rho = \frac{\lambda}{c\mu} = \frac{15.0398}{3*3.3092} = 1.5149$$

Since the traffic intensity is greater than one the queue will grow without bound.

3.4 Probability that there is no outpatient in the system (P_0):

$$p_{o=}\left[\sum_{n=0}^{c-1}\frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^c}{c!\left(1-\frac{\lambda}{c\mu}\right)}\right] = 0.08634$$

3.5 Probability that there is n outpatient in the system (P_n):

$$p_n = \begin{cases} \dfrac{\left(\frac{\lambda}{\mu}\right)^n}{n!} * p_o & \text{if } n \prec c \prec n \\ \dfrac{\lambda^n}{c!\mu^n c^{n-c}} * p_0 & \text{if } c \le n \end{cases}$$

After calculating, we get the following results:

Table 4: Probability that there is n outpatient in the system

n	**P_n**	**n**	**P_n**	**n**	**P_n**
0	0.08634289	3	0.148943899	7	0.367247981
1	0.20054347	4	0.100656877	8	0.012357239
2	0.20965423	5	0.060997345	9	-

3.6 Average number of patient waiting in the queue (L_q):

This is the average number of patients on the queue waiting for a medical doctor. This number was obtained using the following formula:

$$L_q = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} * p_0$$

After calculations, we found that $L_q = 6.0498$ which means that at outpatient department of KIBAGABAGA hospital, we can expect **6 patients** waiting on the door to meet a medical doctor.

3.7 Average number of patient waiting in the system (L):

Average number of patient waiting in the system combines the number of patients waiting on the queue before meeting a medical doctor (Physician) and the system utilization factor of the system.

$$L = L_q + \frac{\lambda}{\mu}$$

After replacing the respective values, we get L=10.5946 which means that at the outpatient department of KIBAGABAGA Hospital, we can expect **11 patients** including those who are waiting on the queue and those who are in the consultation room with the physician.

3.8 Average time a patient spend waiting in the queue:

The time a patient spend waiting on the queue varies from a system to another because when a patient arrives at outpatient department and find all physician are busy, he/she needs to wait for a certain period of time.

Waiting time can be found using the following formula:

$$W_q = \frac{L_q}{\lambda}$$

After replacing, $W_q = 24.13942$ this means that at an outpatient department of KIBAGABAGA Hospital, a patient can wait **24min** before meeting a medical doctor.

3.9 Average time a patient spend waiting in the system:

This corresponds to the time a patient spends on the queue before meeting a medical doctor and a time a patient spends in the consultation room with a medical doctor. The following has been used to calculate the corresponding values:

$$W = \frac{L}{\lambda} = \frac{10.5846}{15.0398} = 0.7044 \approx 42\text{min}$$

This proves that at outpatient department of KIBAGABAGA Hospital, including a time a patient spend in the consultation room with the doctor and the time spend on the queue before meeting a doctor, a farmer can spend almost **42min** on the queue.

3.10 Correlation analysis of waiting line:

Correlation analysis will help us to measure the association between numbers of patients' arrival and days of the week. Fig 1 below illustrates the average variation of arrivals and customers served from Monday to Friday.

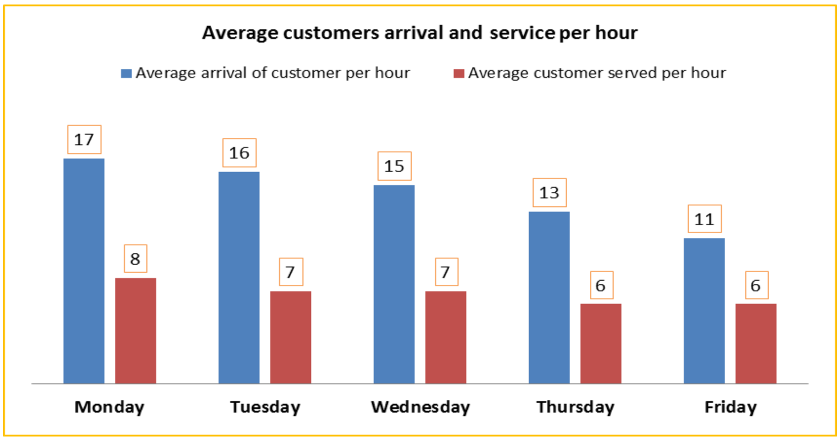

Fig1: Average customers arrival and service per hour

From the above figure, it is clear that the average number of arrivals per hour have been decreasing from the Monday to Friday (17 Patients to 11 Patients respectively), while the average number of service per hour have been almost stable the whole week. Though, we cannot conclude saying that there is an association between patient arrival and working day rather we need to go deep and measure this association.

3.10.1 Correlation between working days of the week and patient arrivals:

Correlation between working days of the week and patient arrivals was calculated using SPSS software under the following null hypothesis $\mathbf{H_0}$: *There is no correlation between working days of the week and patient arrivals*, SPSS displays the following:

Table 5: Pearson correlation coefficient of working days of the week and Arrivals

		Arrivals	Days of the week
Arrivals	Pearson Correlation	1	-.933*
	Sig. (2-tailed)		0.021
	N	5	5
Days of the week	Pearson Correlation	-.933*	1
	Sig. (2-tailed)	0.021	
	N	5	5
*. Correlation is significant at the 0.05 level (2-tailed).			

H_0 will be rejected due to the fact that P-value is less than level of significant ($0.02 \leq 0.05$) which means than there is a significant negative correlation between days and patient arrivals. In another words, there are many patients on Monday than on Friday

3.10.2 Regression Analysis of Patients' Arrivals over the Days of the Week:

Using two variables X and Y, where X represent working days of the week as independent variable and Y stands for average number of arrival as dependent variable, we can undertake a regression analysis

$$Y = \alpha + \beta X$$

The significance of the coefficients has been tested at 95% confidence interval under the following null hypothesis:

$$H_0 = \alpha = 0$$
$$H_0 = \beta = 0$$

We have found the following results:

Table 6: Significance test of the regression model

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	19.500	1.256		15.530	.001
	Days of the week	-1.700	.379	-.933	-4.490	.021
a. Dependent Variable: Arrivals						

Regression model can be formed as follow: $Y=19.5-1.7X$

We can conclude that the model is statistically significant since P-values for α and β are relatively small compared to 0.05 (0.001 and 0.021 respectively). This model can be useful to predicting the number of patients arrived at the outpatient department from Monday to Friday and from this, KIBAGABAGA Hospital can decide how to use efficiently the available resources.

3.10.3 Model validation:

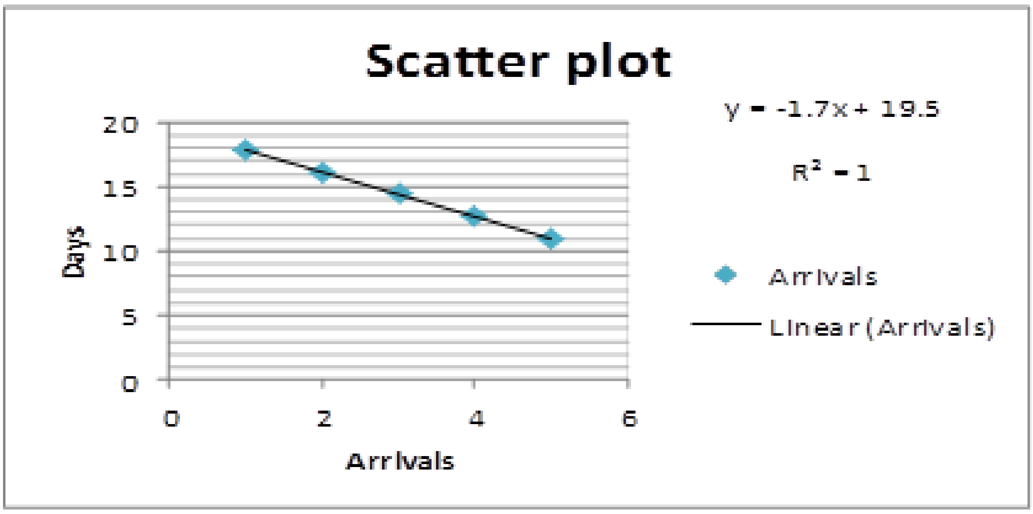

Figure 2: Scatter plot: correlation between X and Y

In the plot above, the straight line comfortably fits thought the data. Hence, there is a strong negative linear relationship between X and Y due to the fact that large values of X correspond to small values of Y

IV. CONCLUSION

The overall objective for this study was to apply a queuing model in KIBAGABAGA Hospital. The findings show that, service utilization factor is greater than one which proves that the queue will grow without bound. There was a large number of patients wait in the queue before meeting a medical doctor and the study revealed that a patient wait for a long time to get served and they are more on Monday than on Friday as proven by correlation analysis which shows that there is a significant negative correlation between working days and patients arrival.

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